Aflevering i uge 7

Opgave 9.2.12

Det er givet følgende 3×4-matrix:

$$\mathsf{A} = \begin{pmatrix} 1 & 1+i & i & 1-i \\ 1+i & 2+i & 1-i & 1 \\ 1+i & 1+4i & 1+3i & 2-i \end{pmatrix}.$$

Idet jeg skal finde rangen af A, vil jeg nu reducere denne til rækkeechelonform. Jeg får, at

$$\begin{aligned}
\mathsf{A} = \begin{pmatrix} 1 & 1+i & i & 1-i \\ 1+i & 2+i & 1-i & 1 \\ 1+i & 1+4i & 1+3i & 2-i \end{pmatrix} &\sim \begin{pmatrix} 1 & 1+i & i & 1-i \\ 1+i & 2+i & 1-i & 1 \\ 0 & -1+3i & 4i & 1-i \end{pmatrix} \\
\sim \begin{pmatrix} 1 & 1+i & i & 1-i \\ 1 & \frac{3}{2}-\frac{1}{2}i & -i & \frac{1}{2}-\frac{1}{2}i \\ 0 & -1+3i & 4i & 1-i \end{pmatrix} &\sim \begin{pmatrix} 1 & 1+i & i & 1-i \\ 0 & \frac{1}{2}-\frac{3}{2}i & -2i & -\frac{1}{2}+\frac{1}{2}i \\ 0 & -1+3i & 4i & 1-i \end{pmatrix} \\
\sim \begin{pmatrix} 1 & 1+i & i & 1-i \\ 0 & 1 & \frac{6}{5}-\frac{2}{5}i & -\frac{2}{5}-\frac{1}{5}i \\ 0 & -1+3i & 4i & 1-i \end{pmatrix} &\sim \begin{pmatrix} 1 & 1+i & i & 1-i \\ 0 & 1 & \frac{6}{5}-\frac{2}{5}i & -\frac{2}{5}-\frac{1}{5}i \\ 0 & 1+i & \frac{8}{5}+\frac{4}{5}i & -\frac{1}{5}-\frac{3}{5}i \end{pmatrix} \\
\sim \begin{pmatrix} 1 & 0 & -\frac{8}{5}+\frac{1}{5}i & \frac{6}{5}-\frac{2}{5}i \\ 0 & 1 & \frac{6}{5}-\frac{2}{5}i & -\frac{2}{5}-\frac{1}{5}i \\ 0 & 1+i & \frac{8}{5}+\frac{4}{5}i & -\frac{1}{5}-\frac{3}{5}i \end{pmatrix} &\sim \begin{pmatrix} 1 & 0 & -\frac{8}{5}+\frac{1}{5}i & \frac{6}{5}-\frac{2}{5}i \\ 0 & 1 & \frac{6}{5}-\frac{2}{5}i & -\frac{2}{5}-\frac{1}{5}i \\ 0 & 1 & \frac{6}{5}-\frac{2}{5}i & -\frac{2}{5}-\frac{1}{5}i \end{pmatrix} \\
\sim \begin{pmatrix} 1 & 0 & -\frac{8}{5}+\frac{1}{5}i & \frac{6}{5}-\frac{2}{5}i \\ 0 & 1 & \frac{6}{5}-\frac{2}{5}i & -\frac{2}{5}-\frac{1}{5}i \\ 0 & 0 & 0 & 0 \end{pmatrix}&.
\end{aligned}$$

Eftersom denne nye matrix har to pivotrækker, er $\operatorname{rang}(\mathsf{A}) = 2$.